

参考答案

第一章

习题 1-1

1. $\sin x = -\frac{6\sqrt{61}}{61}$ $\cos x = \frac{5\sqrt{61}}{61}$ $\tan x = -\frac{6}{5}$
 $\cot x = -\frac{5}{6}$ $\sec x = \frac{\sqrt{61}}{5}$ $\csc x = -\frac{\sqrt{61}}{6}$
2. $\cos x = -\frac{3}{5}$ $\tan x = -\frac{4}{3}$ $\cot x = -\frac{3}{4}$ $\sec x = -\frac{5}{3}$ $\csc x = \frac{5}{4}$.
3. $\cos x = \pm \frac{m\sqrt{1+m^2}}{1+m^2}$.
4. (1) $-\cos 100^\circ$; (2) $\pm \tan x$.
6. $-2 \tan \alpha$.

习题 1-2

1. $\cos(x+y) = -\frac{140}{221}$, $\sin(x-y) = \frac{21}{221}$;
3. 0.96, -0.28, -3.43; 4. $\frac{7}{6}$; 5. 0; 6. 0.75.

习题 1-3

1. $35+80i$; 2. i ; 3. $-\frac{1}{5}+\frac{2}{5}i$; 4. $a=-1$;
5. 400; 6. $\sqrt{2}\left[\cos\left(2x-\frac{\pi}{12}\right)+i\sin\left(2x-\frac{\pi}{12}\right)\right]$.

测试题一

一、判断题

1. $\times$; 2. $\times$; 3. $\times$; 4. $\checkmark$; 5. $\times$; 6. $\times$.

二、填空题

1. 0; 2. $\frac{3}{5}$; 3. $\frac{15\sqrt{2}-8}{34}$; 4. $\sqrt{2}\cos x$; 5. $4\sqrt{3}-4i$;
 6. $\cos 5x - i\sin 5x$; 7. $-8-8\sqrt{3}i$; 8. $64i$.

三、选择题

1. D; 2. C; 3. A; 4. A; 5. B; 6. A.

四、解答题

1. (1) 1; (2) 4.

2. $\cot x$.

3. $\cos 105^\circ = \frac{\sqrt{2}-\sqrt{6}}{4}$; $\cos 15^\circ = \frac{\sqrt{6}+\sqrt{2}}{4}$.

4. 1.

5. $\frac{3}{4}$.

6. $\pm(4+3i)$.

7. $-\frac{9}{61} + \frac{38}{61}i$.

8. 化成三角形式后, 利用复数的相等可得 $\begin{cases} n=2m \\ m=\frac{3}{2}k \end{cases}$, 当 $k=2$ 时, m, n 分

别取最小值 3 和 6.

9. 第三个顶点对应的复数为 $2+\sqrt{3}i$ 或 $\frac{1}{2}-\frac{\sqrt{3}}{2}i$.

10. 提示: 令 $z = \cos \frac{\pi}{11} + i\sin \frac{\pi}{11}$, 则 $z, z^3, \dots, z^9$ 成等比数列, 求出等

比数列的和后, 再利用复数的相等证明之.

第二章

习题 2-1

1. (1) $x \neq 0$ 且 $x \neq 1$; (2) $x \in (-1, 1)$
 (3) $x \in [1, 5]$; (4) $x \in [2k\pi, 2k\pi + \pi] (k \in \mathbf{Z})$.
 2. (1) 偶; (2) 非奇非偶; (3) 偶; (4) 非奇非偶.

3. (1) $T = \pi$; (2) $T = 4\pi$; (3) $T = 2\pi$; (4) 不是周期函数.

4. (1) $y = \frac{x^3+1}{2}$; (2) $y = \frac{1-x}{1+x}$;

(3) $y = \ln \frac{1+x}{1-x}$; (4) $y = \sqrt[3]{1-x^2}$.

5. (1) $y = \sqrt[3]{u}, u = 2x-1$; (2) $y = e^u, u = \sqrt{v}, v = x-2$;

(3) $y = \arccos u, u = 1-x^2$; (4) $y = \sin u, u = e^v, v = -x$;

(5) $y = \ln u, u = 3x^2+2$; (6) $y = \ln u, u = \ln v, v = k^4, k = \ln x$.

6. (1) $y = \cos^3(x+2)$; (2) $y = \sqrt{\cos x}$.

7. 200

8. 44900, 224.5

习题 2-2

1. (1) 0; (2) 0; (3) 0; (4) 1; (5) 0; (6) $\frac{3}{2}$; (7) 0; (8) $\frac{\pi}{4}$.

2. 不存在; 3. -2 4. $a=1$

习题 2-3

1. 是无穷小

(1) $x \rightarrow -\infty$; (2) $x \rightarrow \infty$; (3) $x \rightarrow k\pi (k \in \mathbf{Z})$; (4) $x \rightarrow -1$.

是无穷大

(1) $x \rightarrow +\infty$; (2) $x \rightarrow -1$;

(3) $x \rightarrow k\pi + \frac{\pi}{2} (k \in \mathbf{Z})$; (4) $x \rightarrow -2$ 或 $x \rightarrow +\infty$

2. (1) 0; (2) 0; (3) ∞ ; (4) ∞ ; (5) ∞ ;

(6) 0; (7) ∞ ; (8) 0; (9) 0; (10) 0.

3. 当 $x \rightarrow 0$ 时, $x^2 - x^3$ 是比 $2x - x^2$ 高阶的无穷小.

4. (1) 同阶, 不等价; (2) 等价.

6. (1) $\frac{3}{2}$; (2) $\frac{1}{2}$; (3) $\lim_{x \rightarrow 0} \frac{\sin x^n}{(\sin x)^m} = \begin{cases} 0, & m < n \\ 1, & m = n \\ \infty, & m > n \end{cases}$

(4) 2; (5) $-\sqrt{2}$; (6) e^{-1} ; (7) $\frac{n^2 - m^2}{2}$; (8) $\frac{1}{2}$.

习题 2-4

1. (1) 9; (2) 0; (3) 7; (4) 4.
 2. (1) 2; (2) 0; (3) ∞ ; (4) -1;
 (5) $-\frac{1}{16}$; (6) $-\frac{1}{x^2}$; (7) 2; (8) $\frac{1}{2}$.

习题 2-5

- (1) $\frac{2}{5}$; (2) 9; (3) 2; (4) 1;
 (5) e^{-3} ; (6) e^3 ; (7) e^6 ; (8) e .

习题 2-6

1. 函数 $f(x)$ 在 $x = \frac{1}{2}$ 处连续, 在 $x = 1$ 处间断, 在 $x = 2$ 处连续.
 2. $(-\infty, -1) \cup (-1, 2) \cup (2, +\infty)$, $\frac{2}{3}$, 0, ∞ .
 3. (1) $x = 1$; (2) $x = -3$, $x = 2$;
 (3) $x = -2$; (4) $x = 0$ $x = k\pi (k \in \mathbf{Z}, k \neq 0)$;
 (5) $x = 1$, $x = -1$; (6) $x = 1$.
 4. (1) $\sqrt{5}$; (2) $\frac{1-e^{-2}}{2}$; (3) $\frac{\sqrt{2}}{2}$; (4) $\frac{1}{2}$; (5) -2;
 (6) 2; (7) $2\sqrt{2}$; (8) $\frac{1}{2}$; (9) $\frac{\pi}{2}$; (10) 0.
 5. 1; -1.
 6. e^4 ; e^2 .

测试题二

一、判断题

1. $\times$ 2. $\times$ 3. $\times$ 4. $\times$ 5. $\times$.

二、填空题

1. $y = e^u$, $u = \cos v$, $v = 2x + 1$
 2. $\frac{\sqrt{x}}{2x}$ 3. e^{-1} 4. $\frac{1}{3}$ 5. 0

三、选择题

1. A 2. D 3. A 4. C 5. A

四、解答题

1. (1) $y = \sqrt{u}$, $u = \arctan v$, $v = x + 1$;

(2) $y = u^2$, $u = \tan v$, $v = \frac{x}{2}$.

2. (1) $\frac{4}{3}$; (2) $\frac{1}{2}$; (3) $e^{-\frac{1}{2}}$;

(4) $-\frac{1}{2}$; (5) 0; (6) $\frac{1}{2}$.

3. 连续.

5. $C(x) = 40000 + 4250x$ ($0 \leq x \leq 100$).

6. (1) 44; (2) 210.

第三章

习题 3-1

1. (1) 6; (2) -2.

2. $v(3) = 36$.

3. 切线方程为 $x - 2y + \sqrt{3} - \frac{\pi}{3} = 0$; 法线方程为 $2x + y - \frac{\sqrt{3}}{2} - \frac{2\pi}{3} = 0$.

4. $(2, \ln 2)$.

5. $C'(Q) = 4 + 0.1Q$, $C'(200) = 24$.

6. (1) 连续, 不可导; (2) 连续且可导.

7. $a = 2$, $b = -1$.

习题 3-2

1. (1) $-\frac{1}{x^2} - \frac{1}{2\sqrt{x}}$;

(2) $e^x \cos x - e^x \sin x$;

(3) $y = \frac{1 - x \ln 4}{4^x}$;

(4) $\frac{1}{2\sqrt{x}} \ln x \cos x + \frac{1}{\sqrt{x}} \cos x - \sqrt{x} \ln x \sin x$;

(5) $\frac{e^x(1-x)^2}{(1+x^2)^2}$;

(6) $2x \arctan x + \frac{x^2}{1+x^2} - \frac{1}{x}$;

$$(7) \csc x \left(\frac{1}{2\sqrt{x}} - \sqrt{x} \cot x \right); \quad (8) 5^x \ln 5 + 6x^5.$$

$$2. f' \left(\frac{\pi}{4} \right) = 1 - \frac{\sqrt{2}}{4} + \frac{\pi}{2}.$$

$$3. (1) e^{\sin x} \cos x; \quad (2) \frac{6x}{\sqrt{1-9x^4}}; \quad (3) -12x(1-x^2)^5;$$

$$(4) y = \frac{-2x}{\sqrt{1-2x^2}}; \quad (5) \frac{-\tan \sqrt{x}}{2\sqrt{x}}; \quad (6) -\frac{1}{x^2} \sec^2 \frac{1}{x};$$

$$(7) -e^{-3x}(3\cos 2x + 2\sin 2x); \quad (8) \sec x;$$

$$(9) \frac{1}{(1-x^2)^{\frac{3}{2}}}; \quad (10) \frac{1}{x \ln x \ln(\ln x)}.$$

习题 3-3

$$1. (1) y' = \frac{e^x - y}{x + e^y}; \quad (2) y' = \frac{y \cos x + \sin(x-y)}{\sin(x-y) - \sin x};$$

$$(3) y' = \frac{1+y^2}{2+y^2}; \quad (4) y' = -\frac{2\sin 2x + ye^{xy} + \frac{y}{x}}{xe^{xy} + \ln x}.$$

$$2. y'|_{x=0} = \frac{3}{2}.$$

$$3. 5x - 4y - 1 = 0.$$

$$4. (1) y' = (x+1)^{\sin x} \left[\cos x \ln(1+x) + \frac{\sin x}{1+x} \right];$$

$$(2) y' = \left(\cos x \frac{1}{2x} \right)^{2x} \left[2 \ln \cos \frac{1}{2x} + \frac{1}{x} \tan \frac{1}{2x} \right];$$

$$(3) y' = x^x [\ln x + 1];$$

$$(4) y' = \frac{\sqrt{x+1}(2-x)^2}{(2x-1)^3} \left[\frac{1}{2(x+1)} + \frac{2}{x-2} - \frac{6}{2x-1} \right].$$

$$5. (1) y' = -\frac{2}{3} e^{2t}; \quad (2) y' = \frac{\cos t - \sin t}{\cos t + \sin t};$$

$$(3) y' = \frac{3t}{2} - \frac{1}{2t}; \quad (4) y' = \frac{\cos \theta - \theta \sin \theta}{1 - \sin \theta - \theta \cos \theta}.$$

习题 3-4

1. (1) $4 - \frac{1}{x^2}$; (2) $4e^{2x-1}$;
 (3) $-\frac{1}{\sqrt{(x^2-1)^3}}$; (4) $-\frac{16x}{(1+4x^2)^2}$.
 2. (1) $(-1)^{n-1} \frac{(n-1)!}{(1+x)^n}$; (2) $e^x(x+n)$; (3) $2^{n-1} \sin[2x + \frac{\pi}{2}(n-1)]$.

习题 3-5

1. (1) $dy = \frac{2\ln(1-x)}{x-1} dx$;
 (2) $dy = 2^{\ln(\tan x)} \csc x \cdot \sec x \ln 2 dx$;
 (3) $dy = 4x \sec^2(1+x^2) \tan(1+x^2) dx$;
 (4) $dy = \frac{x^2 \sin x - \sin x + 2x \cos x}{(1-x^2)^2} dx$.
 2. (1) $-5x + C$; (2) $\frac{3}{2}x^2 + C$;
 (3) $-\frac{e^{-2x}}{2} + C$; (4) $\ln|x-2| + C$;
 (5) $\arcsin 2x + C$; (6) $\tan 2x + C$;
 (7) $\frac{1}{1+25x^2}$; (8) $5^{x^3} \cdot \ln 5$, $5^{x^3} \cdot 3x^2 \cdot \ln 5$.
 3. (1) -0.8617 (2) 0.8104 (3) 0.985 (4) -0.02

测试题三

一、填空题

1. $2\sqrt{x} + C$, $\frac{1}{3}e^{3x+1} + C$ 2. 0 3. $f''(0) = -2$
 4. 2.6367 5. n

二、选择题

1. B 2. C 3. C 4. D 5. A

三、选择正确的求导方法求下列函数的导数

1. $\sin x + \sin x \ln x + x \cos x \ln x$ 2. $-6xe^{-3x^2}$

3. $3x^2 \ln x + x^2 - 2 \sin x$

4. $\frac{7}{8}x^{-\frac{1}{8}}$

5. $\frac{1}{x-2}$

6. $4 \cos(2x+7)$

7. $-12 \cos^2 4x \sin 4x$

8. $\frac{\sin 3x}{x} + 3 \ln(2x) \cdot \cos(3x)$

9. $2x \cos 3x \cdot \sec^2 x^2 - 3 \sin 3x \cdot \tan x^2$

10. $\frac{1}{2x\sqrt{6x-1}}$

11. $y' = \frac{(x+1)^2 \sqrt{x-3}}{(3x+1)^3} \left[\frac{2}{x+1} + \frac{1}{2x-6} - \frac{9}{3x+1} \right]$

12. $y' = (\sin x)^{\cos x} \left[\frac{-\sin^2 x \cdot \ln(\sin x) + \cos^2 x}{\sin x} \right]$

四、求由下列方程所确定的隐函数 y 的导数

1. $y' = \frac{y}{y-1}$

2. $y' = \frac{1-x+y-y(y-x)\cos(xy)}{1+x(y-x)\cos(xy)}$

五、解答题

1. $v(t) = gt$ 2. 6 3. $3x-12y+1=0$, $3x-12y-1=0$

5. $f'(x) = 3x^2 - 4x + 5$, $f'(-1) = 12$

第四章

习题 4-1

1. (1) $\xi = 0$; (2) $\xi = \frac{8}{3}$; (3) $\xi = \frac{\pi}{2}$; (4) 不满足.

2. (1) $\xi = \frac{1}{\ln 2}$; (2) $\xi = \frac{5 \pm \sqrt{13}}{12}$.

习题 4-2

(1) $\cos \alpha$; (2) 1; (3) $\frac{1}{2}$; (4) 1; (5) $\frac{1}{2}$;

(6) 0; (7) 0; (8) 2; (9) 1; (10) 1.

习题 4-3

1. (1) 单增区间是 $(\frac{1}{2}, +\infty)$, 单减区间是 $(0, \frac{1}{2})$;
(2) 单增区间是 $(-\infty, -1)$ 和 $(1, +\infty)$, 单减区间是 $(-1, 1)$;
(3) 单增区间是 $(0, +\infty)$, 单减区间是 $(-\infty, -1)$ 和 $(-1, 0)$;
(4) 单增区间是 $(-\infty, -\frac{3}{5})$ 和 $(1, +\infty)$, 单减区间是 $(-\frac{3}{5}, 1)$.

习题 4-4

1. (1) 极小值 $f(0)=0$;
(2) 极小值 $f(3)=-47$, 极大值 $f(-1)=17$;
(3) 极大值 $f(2)=1$;
(4) 极小值 $f(0)=0$;
(5) 极小值 $f(0)=0$, 极大值 $f(2)=4e^{-2}$;
(6) 极小值 $f(-1)=-1$, 极大值 $f(1)=1$.
2. (1) 极小值 $f(0)=0$, 极大值 $f(\frac{2}{3})=\frac{4}{27}$; (2) 极小值 $f(1)=2-4\ln 2$
3. $a=2e^2$.

习题 4-5

1. (1) 最大值 $f(4)=138$, 最小值 $f(1)=3$;
(2) 最大值 $f(0)=\frac{\pi}{4}$, 最小值 $f(1)=0$;
(3) 最大值 $f(-1)=\frac{7}{2}$, 最小值 $f(1)=-\frac{5}{2}$;
(4) 最大值 $f(\frac{3}{4})=\frac{5}{4}$, 最小值 $f(-5)=\sqrt{6}-5$.
2. $r=\sqrt[3]{\frac{V}{2\pi}}$, $h=2r$.
3. $x=\frac{a}{2}$.
4. 5 小时.
5. $Q=4$.

6. $x=30$ ，最低的平均费用是 80.

7. 长 $18\sqrt{2}$ 米，宽 $6\sqrt{2}$ 米.

习题 4-6

1. (1) 凸区间为 $(0,1)$ ，凹区间为 $(-\infty,0)$ 和 $(1,+\infty)$ ，拐点为 $(0,1)$ 和 $(1,0)$ ；

(2) 凸区间为 $(-\infty,2)$ ，凹区间为 $(2,+\infty)$ ，拐点为 $(2,2e^{-2})$ ；

(3) 凸区间为 $(-\infty,-1)$ 与 $(1,+\infty)$ ，凹区间为 $(-1,1)$ ，拐点为 $(-1,\ln 2)$ 与 $(1,\ln 2)$ ；

(4) 凸区间为 $(4,+\infty)$ ，凹区间为 $(-\infty,4)$ ，拐点为 $(4,2)$.

2. (1) 水平渐近线 $y=0$ ，垂直渐近线 $x=5$ ；

(2) 水平渐近线 $y=1$ ，垂直渐近线 $x=0$ ；

(3) 水平渐近线 $y=1$ ，垂直渐近线 $x=2$ ， $x=-3$ ；

(4) 水平渐近线 $y=-2$ ，垂直渐近线 $x=1$ ， $x=-1$.

习题 4-8

1. 总收益 $R(30)=120$ ，平均收益 $\bar{R}(30)=4$ ，边际收益 $R'(30)=-2$.

2. $C'(200)=24$. 经济意义：可以近似理解为在产量为 200 单位时，当产量变化一个单位时，成本变动 24 元.

3. $\eta(p)=-2p$

4. $\eta(p)=\frac{p}{10}$

$\eta(5)=0.5$ ，说明 $p=5$ 时，价格上涨 1%，需求量减少 0.5%；

$\eta(10)=1$ ，说明 $p=10$ 时，价格与需求的变动幅度相同；

$\eta(15)=1.5$ ，说明 $p=15$ 时，价格上涨 1%，需求量减少 1.5%.

5. (1) $\eta(p)=-\frac{p}{24-p}$ ； (2) $\eta(p)=-\frac{1}{3}$ ；

(3) 当 $p=6$ 时，价格 p 上涨 1%，总收益约增加 0.67%.

测试题四

一、填空题

1. $f(b)$ ， $f(a)$ 2. $a=0$ ， $b=-3$

3. $f'(\xi)=\frac{f(b)-f(a)}{b-a}$ ($a<\xi<b$)

4. 驻点(或稳定点) 5. (1, -1)

6. $R(Q) = 10Q - \frac{Q^2}{5}$, $R'(Q) = 10 - \frac{2Q}{5}$

7. -6

二、选择题

1. D 2. D 3. D 4. B 5. C

三、求下列极限

1. 1 2. $-\frac{5}{3}$ 3. 0 4. -1 5. 1 6. 1

四、解答题

1. $\xi = \frac{3}{2}$ 2. $\xi = \frac{2}{3}\sqrt{3}$

3. 极小值 $f(2) = -48$, 极大值 $f(-4) = 60$.

4. 极小值 $f(\frac{2}{5}) = -\frac{3}{25}\sqrt[3]{20}$, 极大值 $f(0) = 0$.

单增区间是 $(-\infty, 0)$ 和 $(\frac{2}{5}, +\infty)$, 单减区间是 $(0, \frac{2}{5})$.

5. $a = 2$, 极大值 $f(\frac{\pi}{3}) = \sqrt{3}$.

6. (1) 凹区间为 $(-\infty, +\infty)$; 无拐点;

(2) 凸区间为 $(-\infty, \frac{1}{2})$; 凹区间为 $(-\frac{1}{2}, 0)$, $(0, 1)$, $(1, +\infty)$; 拐点为 $(-\frac{1}{2}, -3\sqrt[3]{2})$.

7. (1) 垂直渐近线 $x = 0$;

(2) 水平渐近线 $y = 0$, 垂直渐近线 $x = 2$, $x = -2$.

五、应用题

1. 250 2. 宽为 $\frac{30}{\pi+4}$ 米, 高为 $\frac{15}{\pi+4}$ 米 3. -8, 约-0.54

第五章

习题 5-1

1. (1) $\frac{2}{7}x^{\frac{7}{2}} + C$;

(2) $\frac{3}{2}x^{\frac{2}{3}} - \frac{3}{5}x^{\frac{5}{3}} + C$;

$$(3) \ln|x| + \frac{2^x e^x}{\ln|2e|} + \tan x + C; \quad (4) \frac{2}{5}x^{\frac{5}{2}} + \frac{1}{2}x^2 + 6x^{\frac{1}{2}} + C;$$

$$(5) x - \arctan x + C; \quad (6) \frac{10^x}{\ln 10} - \cot x - x + C;$$

$$(7) \frac{2^x}{\ln 2} - \frac{\left(\frac{2}{3}\right)^x}{\ln 2 - \ln 3} + C; \quad (8) \tan x - \cot x + C;$$

$$(9) \tan x - \sec x + C; \quad (10) \frac{1}{2}(x - \sin x) + C;$$

$$(11) \sin x - \cos x + C; \quad (12) e^x - x + C;$$

$$(13) \frac{1}{2} \tan x + C; \quad (14) -2 \arcsin x + C;$$

$$2. s = 2 \sin t + s_0.$$

$$3. y = -\frac{1}{2}x^2 + 2x + 3.$$

$$4. C = x^3 - 2x^2 + 100x + 10000$$

习题 5-2

$$1. (1) -\frac{1}{2} \ln|1-2x| + C; \quad (2) \frac{1}{3}(2x-1)^{\frac{3}{2}} + C;$$

$$(3) \frac{1}{3}e^{3x+1} + C; \quad (4) -\frac{3}{5} \cos \frac{5x}{3} + C;$$

$$(5) -\frac{1}{2} \sin(1-2x) + C; \quad (6) \frac{1}{7} \tan 7x + C;$$

$$(7) -\sqrt{1-x^2} + C; \quad (8) \ln(1+e^x) + C;$$

$$(9) \frac{1}{3} \ln^3 x + C; \quad (10) \frac{1}{3} \ln(1+x^3) + C;$$

$$(11) -\cos x + \frac{1}{3} \cos^3 x + C; \quad (12) \tan x + \frac{2}{3} \tan^3 x + \frac{1}{5} \tan^5 x + C;$$

$$(13) \frac{1}{3} \sin^3 x - \frac{2}{5} \sin^5 x + \frac{1}{7} \sin^7 x + C; \quad (14) \frac{1}{2} \ln(1+e^{2x}) + C;$$

$$(15) \frac{1}{3} \arctan^3 x + C; \quad (16) -2 \cos \sqrt{x} + C.$$

$$2. (1) 2\sqrt{x-1} - 2 \arctan \sqrt{x-1} + C;$$

$$(2) \sqrt{2x-3} - \ln(1+\sqrt{2x-3}) + C;$$

$$(3) 2\sqrt{x} - 3\sqrt[3]{x} + 6\sqrt[6]{x} - 6\ln(\sqrt[6]{x} + 1) + C;$$

$$(4) \frac{1}{15}\sqrt[3]{(3x+1)^5} + \frac{1}{3}\sqrt[3]{(3x+1)^2} + C;$$

$$(5) 2\arcsin \frac{x}{2} - \frac{x}{2}\sqrt{4-x^2} + C;$$

$$(6) \sqrt{x^2-2} - \sqrt{2} \arccos \frac{\sqrt{2}}{x} + C.$$

习题 5-3

$$(1) e^x(x^2 - 2x + 2) + C;$$

$$(2) \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C;$$

$$(3) x \sin x + \cos x + C;$$

$$(4) x \arcsin x + \sqrt{1-x^2} + C;$$

$$(5) -xe^{-x} - e^{-x} + C$$

$$(6) -\frac{\ln x}{x} - \frac{1}{x} + C$$

$$(7) \frac{1}{2}e^{-x}(\sin x - \cos x) + C;$$

$$(8) -\frac{1}{4}x \cos 2x + \frac{1}{8} \sin 2x + C;$$

$$(9) \frac{1}{2}(x^2 - 1)\ln(x-1) - \frac{1}{4}x^2 - \frac{1}{2}x + C; \quad (10) x \tan x + \ln|\cos x| - \frac{x^2}{2} + C.$$

习题 5-4

$$(1) \ln \left| \frac{x+2}{x+3} \right| + C;$$

$$(2) \ln |(x-7)^4(x+1)| + C;$$

$$(3) \frac{1}{2} \ln \left| \frac{x-1}{x+1} \right| - \frac{1}{x+1} + C;$$

$$(4) -\ln|x-1| + \frac{3}{2} \ln|x^2-2x+5| + \frac{1}{2} \arctan\left(\frac{x-1}{2}\right) + C;$$

$$(5) \ln|x^2+2x+2| + \arctan(x+1) + C;$$

$$(6) \frac{1}{2}x^2 + x + \ln \frac{(x-3)^6}{|x-2|^5} + C.$$

测试题五

一、填空题

$$1. \frac{x}{1+\ln x} dx$$

$$2. \frac{1}{2}x^2 + x \ln x - x + C$$

$$3. \cos \frac{x}{2}$$

$$4. \frac{1}{2} \ln|x^2+2|+C \quad 5. -f\left(\frac{1}{x}\right)+C$$

二、选择题

$$1. D \quad 2. B \quad 3. D \quad 4. C \quad 5. A$$

三、求下列函数的不定积分

$$1. y = \frac{2}{3}x^{\frac{3}{2}} - 2x + C$$

$$2. \tan x + \cot x + C$$

$$3. \frac{3}{4}(1+\ln x)^{\frac{4}{3}} + C$$

$$4. \sqrt{x^2+4} + C$$

$$5. 2\sqrt{e^x-1} - 2\arctan\sqrt{e^x-1} + C$$

$$6. \sqrt{x} - \arctan\sqrt{x} + C$$

$$7. 2x\sin\frac{x}{2} + 4\cos\frac{x}{2} + C$$

$$8. x\ln(x^2+1) - 2x + 2\arctan x + C$$

$$9. \ln|x| - \ln|x-1| - \frac{1}{x-1} + C$$

$$10. \frac{1}{2}\ln|x^2+2x-3| + C$$

$$11. 2e^{\sqrt{x}}(\sqrt{x}-1) + C$$

$$12. \frac{1}{4}\ln\left|\frac{x-3}{x+1}\right| + C$$

四、综合应用题

$$1. y = x - \ln|x| + 1.$$

$$2. s = t^3 + 2t^2.$$

$$3. R(q) = \frac{1}{3}q^3 + 25q^2 + C.$$

第六章

习题 6-1

$$1. \frac{1}{3}(b^3 - a^3) + b - a.$$

$$2. (1) \frac{1}{2}(b^2 - a^2);$$

$$(2) e - 1.$$

$$3. (1) \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x dx - \int_{\frac{\pi}{2}}^{\pi} \cos x dx;$$

$$(2) \int_a^b [f(x) - g(x)] dx;$$

$$(3) 1 - \int_0^1 x^2 dx;$$

$$(4) \int_{-1}^1 (\sqrt{2-x^2} - x^2) dx.$$

$$4. (1) 2;$$

$$(2) 8.$$

$$5. (1) \frac{26}{3}; \quad (2) -\frac{10}{3}.$$

习题 6-2

$$\begin{aligned} 1. (1) 20; \quad (2) a\left(a^2 - \frac{a}{2} + 1\right); \quad (3) \frac{21}{8}; \\ (4) \frac{271}{6}; \quad (5) \frac{\pi}{6}; \quad (6) \frac{\pi}{3}; \\ (7) \frac{\pi}{3a}; \quad (8) \frac{\pi}{6}; \quad (9) \frac{\pi}{4} + 1; \\ (10) -1; \quad (11) 1 - \frac{\pi}{4}; \quad (12) 4; \quad (13) \frac{8}{3}. \end{aligned}$$

$$2. (1) \frac{76}{3}; \quad (2) 2.$$

$$\begin{aligned} 3. (1) 0; \quad (2) \frac{1}{4}; \quad (3) \frac{\pi}{6} - \frac{\sqrt{3}}{8}; \\ (4) \sqrt{2}(\pi + 2); \quad (5) \frac{a^4 \pi}{16}; \quad (6) \frac{1}{6}; \\ (7) 1 - 2\ln 2; \quad (8) 1 - \frac{1}{\sqrt{e}}; \quad (9) \frac{\pi}{2}. \end{aligned}$$

$$4. (1) 0; \quad (2) \frac{3}{2}\pi; \quad (3) 0.$$

$$5. (1) 1 - \frac{2}{e}; \quad (2) -\frac{\pi}{\omega^2}; \quad (3) 4(2\ln 2 - 1);$$

$$(4) \frac{1}{5}(e^\pi - 2); \quad (5) \frac{\pi^3}{6} - \frac{\pi}{4}; \quad (6) 2e - 2.$$

习题 6-3

$$(1) \frac{1}{3}; \quad (2) \text{发散}; \quad (3) \pi; \quad (4) 1;$$

$$(5) \text{发散}; \quad (6) \frac{8}{3}; \quad (7) \frac{\pi}{2}.$$

习题 6-4

$$1. (1) \frac{1}{6}; \quad (2) 1; \quad (3) \frac{32}{3}; \quad (4) \frac{32}{3}.$$

$$2. (1) \frac{4}{3}; \quad (2) \frac{3}{2} - \ln 2; \quad (3) e + \frac{1}{e} - 2;$$

$$(4) b - a; \quad (5) \frac{7}{6}.$$

$$3. (1) \frac{32}{3}\pi; \quad (2) \frac{3}{10}\pi; \quad (3) 160\pi^2.$$

$$4. \frac{128}{7}\pi, \frac{64}{5}\pi. \quad 5. 3.3 \text{ N}. \quad 6. 0.75 \text{ (J)}.$$

$$7. 2.06 \times 10^5 \text{ (N)}. \quad 8. 1.44 \times 10^7 \text{ (N)}. \quad 9. 5.77 \times 10^7 \text{ (J)}.$$

$$10. 7.70 \times 10^7 \text{ (J)}. \quad 11. 1.65 \text{ (N)}. \quad 12. \frac{4.84}{R} \times 10^4 \text{ (W)}.$$

$$13. I = \frac{I_m}{2}. \quad 14. 700 \text{ 元}. \quad 15. 2400 \text{ 元}.$$

测试题六

一、填空题

$$1. \int_a^b g(x) dx \quad 2. 0 \quad 3. 2 \int_0^a f(x) dx$$

$$4. \int_a^b f(x) dx = \int_a^B f[\varphi(t)] \varphi'(t) dt$$

$$5. \int_a^b u dv = [uv]_a^b - \int_a^b v du$$

二、选择题

$$1. A \quad 2. B \quad 3. C \quad 4. D \quad 5. B$$

三、判断题

$$1. \times \quad 2. \times \quad 3. \checkmark \quad 4. \times \quad 5. \times$$

四、计算下列定积分

$$1. 20 \quad 2. \frac{21}{8} \quad 3. \frac{1}{8} \quad 4. 1$$

$$5. 2 + 2 \ln 2 \quad 6. \pi \quad 7. \frac{1}{4}(1 - 3e^{-2}) \quad 8. \frac{1}{3}$$

五、解答题

$$1. \frac{17}{4}. \quad 2. \frac{32}{5}\pi. \quad 3. 1.37 \times 10^5 \text{ (N)}.$$

第七章

习题 7-1

- (1) 一阶; (2) 三阶; (3) 二阶; (4) 一阶.
- (1) 特解; (2) 通解; (3) 不是; (4) 特解.
- (1) $\sqrt{1-x^2}y' - y = 0$;
(2) $y'' - \frac{2}{x}y' + \frac{2}{x^2}y = 0$.

习题 7-2

- (1) $y = \frac{1}{\ln(C\sqrt{1+x^2})}$; (2) $y = Ce^{x^2}$;
(3) $\tan y \tan x = C$; (4) $10^{-y} + 10^x = C$.
- (1) $y^2 - 1 = 2\ln(1+e^x) - 2\ln(1+e)$;
(2) $2(x^3 - y^3) + 3(x^2 - y^2) + 5 = 0$.
- $x^2 + y^2 = 2$.
- $V = \frac{mg}{k}(1 - e^{-\frac{k}{m}t})$ (比例常数 $k > 0$).
- $u_c = u_0 e^{-\frac{t}{RC}}$.

习题 7-3

- (1) $\sin \frac{y}{x} = Cx$; (2) $y = 2x \arctan(Cx)$;
(3) $x - \sqrt{xy} = C$; (4) $y = Ce^{\frac{x^3}{3y^3}}$.
- (1) $y^2 = 2x^2(\ln x + 2)$; (2) $\frac{x+y}{x^2+y^2} = 1$.

习题 7-4

- (1) $y = \frac{1}{3}x^2 + \frac{3}{2}x + 2 + \frac{C}{x}$; (2) $y = C \cos x - 2 \cos^2 x$;

$$(3) \quad y^2 - 2x = Cy^3;$$

$$(4) \quad x = Ce^{-y} + e^y.$$

$$2. (1) \quad y = \frac{\pi - 1 - \cos x}{x};$$

$$(2) \quad y = \frac{2}{3}(4 - e^{-3x});$$

$$(3) \quad y = \frac{1}{2x}[1 + (\ln x)^2].$$

$$3. \quad y = 2(e^x - x - 1).$$

$$4. \quad t = 60 \text{ min.}$$

习题 7-5

$$1. (1) \quad y = \frac{1}{6}x^3 - \sin x + C_1x + C_2;$$

$$(2) \quad y = (x - 3)e^x + C_1x^2 + C_2x + C_3;$$

$$(3) \quad y = C_1 \ln x + C_2;$$

$$(4) \quad y = \arcsin(C_2e^x) + C_1.$$

$$2. (1) \quad y = \sqrt{2x - x^2} \quad (0 < x < 2);$$

$$(2) \quad y = 2e^x - \frac{x^2}{2} - x - 1.$$

$$3. \quad y = \frac{x^3}{6} + \frac{x}{2} + 1.$$

$$4. \quad x = \frac{F_0}{m} \left(\frac{1}{2}t^2 - \frac{1}{6T}t^3 \right) \quad (0 \leq t \leq T).$$

习题 7-6

$$1. \quad y = (C_1 + C_2x)e^{x^2}.$$

$$4. \quad y = C_1x + C_2e^x + x^2.$$

$$5. \quad y_2 = \varphi(x) \int \frac{e^{-\int p dx}}{\varphi^2(x)} dx, \quad y = \varphi(x) [C_1 + C_2 \int \frac{e^{-\int p dx}}{\varphi^2(x)} dx].$$

习题 7-7

$$1. (1) \quad y = C_1e^{-x} + C_2e^{3x};$$

$$(2) \quad y = e^x(C_1 \cos 2x + C_2 \sin 2x);$$

$$(3) \quad y = (C_1 + C_2x)e^{-3x}.$$

$$2. (1) \quad y = C_1e^{\frac{x}{2}} + C_2e^{-x} + e^x;$$

$$(2) y = C_1 + C_2 e^{-\frac{5}{2}x} + \frac{1}{3}x^3 - \frac{3}{5}x^2 + \frac{7}{25}x;$$

$$(3) y = C_1 e^{3x} + C_2 x e^{3x} + \frac{x^2}{2} e^{3x} \left(\frac{1}{3}x + 1 \right).$$

$$3. (1) y = C_1 \cos 2x + C_2 \sin 2x + \frac{1}{3}x \cos x + \frac{2}{9} \sin x;$$

$$(2) y = e^x (C_1 \cos 2x + C_2 \sin 2x) - \frac{1}{4} x e^x \cos 2x;$$

$$(3) y = C_1 e^x + C_2 e^{6x} + \frac{5}{74} \sin x + \frac{7}{74} \cos x.$$

$$4. (1) y = -\frac{7}{6} e^{-2x} + \frac{5}{3} e^x - x - \frac{1}{2};$$

$$(2) y = -\frac{1}{3} \sin x - \cos x + \frac{1}{3} \sin 2x;$$

$$(3) y = -5e^x + \frac{7}{2} e^{2x} + \frac{5}{2}.$$

$$5. i = \frac{1}{202} (101e^{-5t} - 25e^{-t} + 48\sin 10t - 76\cos 10t).$$

$$6. x = \frac{mg}{k} t - \frac{m^2 g}{k^2} (1 - e^{-\frac{k}{m}t}).$$

测试题七

一、填空题

$$1. \text{含有未知函数的导数或微分的方程} \quad 2. \frac{dy}{dx} = f(x)g(x)$$

$$3. y = \ln|x| + C$$

$$4. y = C e^{2x} \quad 5. r^2 - 5r + 6 = 0$$

二、选择题

$$1. C \quad 2. B \quad 3. C \quad 4. A \quad 5. A$$

三、判断题

$$1. \times \quad 2. \times \quad 3. \checkmark \quad 4. \times \quad 5. \checkmark$$

四、求下列微分方程的通解

$$1. y = x^3 + C \quad 2. y = -\frac{1}{3} C e^{\frac{3}{2}x^2}$$

$$3. \quad y = -\sin t + C_1 t^2 + C_2 t + C_3$$

$$4. \quad e^y = Cx^2 y \quad 5. \quad y = \frac{1}{2}(\sin x + \cos x) + Ce^{-x}$$

$$6. \quad y = -e^{-2x} + Ce^{4x}$$

$$7. \quad y = \frac{1}{2}\ln^2 x + C_1 \ln x + C_2 \quad 8. \quad y = C_1 e^{-2x} + C_2 e^x$$

$$9. \quad y = C_1 e^{-3x} + C_2 e^{-x} - \cos x + \frac{1}{2}\sin x$$

$$10. \quad y = C_1 + C_2 e^{-9x} - \frac{1}{5}\cos 3x - \frac{1}{15}\sin 3x$$

五、求下列微分方程的特解

$$1. \quad y = (4 + 26x)e^{-6x}$$

$$2. \quad y = 1 + x^2 + \sqrt{1 + x^2} \left[\ln(x + \sqrt{1 + x^2}) - \frac{1}{2} \right]$$

$$3. \quad y = e^{-x^2} \left(\frac{1}{2}x^2 + 1 \right)$$

$$4. \quad y = e^y [\ln(e^x + 1) - x + 1 - \ln^2 x] - 1$$

$$5. \quad y = 2xe^{-3x} + \frac{5}{6}x^3 e^{-3x}$$

六、解答题

$$1. \quad y = (1 - x)e^{2x}$$

$$2. \quad x = 0.02\cos 10t$$

第八章

习题 8-1

$$1. \quad (1) 10; \quad (2) 5; \quad (3) \sin \alpha \cos \beta - \cos \alpha \sin \beta; \quad (4) 0;$$

$$(5) -260; \quad (6) 8; \quad (7) -9; \quad (8) -2abc.$$

$$2. \quad (1) \begin{cases} x = \frac{2}{7} \\ y = \frac{9}{7} \end{cases}; \quad (2) \begin{cases} x_1 = 6 \\ x_2 = 10 \end{cases};$$

$$(3) \begin{cases} x=1 \\ y=2 \\ z=1 \end{cases}; \quad (4) \begin{cases} x_1 = \frac{1}{2a}(b+c-a) \\ x_2 = \frac{1}{2b}(c+a-b) \\ x_3 = \frac{1}{2c}(a+b-c) \end{cases}.$$

习题 8-2

(1) 164; (2) $3a-b+2c+d$; (3) 70; (4) -2; (5) -24; (6) 312.

习题 8-3

$$1. (1) \begin{cases} x_1=1 \\ x_2=2 \\ x_3=3 \end{cases}; \quad (2) \begin{cases} x_1=3 \\ x_2=-4 \\ x_3=-1 \\ x_4=1 \end{cases}.$$

2. $\lambda = -1$ 或 $\lambda = 4$

习题 8-4

1. $a=0, b=3, c=-3, d=30$

$$2. A+B = \begin{pmatrix} 7 & 5 & 14 \\ 2 & 11 & 2 \\ 10 & 12 & -6 \end{pmatrix}, \quad B-A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 5 & 0 \\ 2 & 2 & -4 \end{pmatrix},$$

$$3A+2B = \begin{pmatrix} 17 & 12 & 35 \\ 5 & 25 & 5 \\ 24 & 29 & -13 \end{pmatrix}, \quad 3A-2B = \begin{pmatrix} 1 & 0 & 7 \\ 1 & -7 & 1 \\ 0 & 1 & 7 \end{pmatrix}.$$

$$3. (1) \begin{pmatrix} 14 & 3 \\ 24 & 5 \end{pmatrix}; \quad (2) = \begin{pmatrix} 4 & 6 & 2 \\ 7 & 14 & 10 \\ 2 & 3 & 1 \end{pmatrix}; \quad (3) (10);$$

$$(4) \begin{pmatrix} -1 & -2 \\ -2 & -4 \\ -3 & -6 \end{pmatrix}; \quad (5) \begin{pmatrix} 2 & -4 \\ 1 & -2 \end{pmatrix}; \quad (6) \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}.$$

$$4. A^T = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 3 & 0 \\ -1 & 2 & 2 \end{pmatrix}, \quad B^T = \begin{pmatrix} 0 & 2 & -1 \\ 1 & -1 & -1 \\ 2 & 0 & 3 \end{pmatrix},$$

$$A^T + B^T = \begin{pmatrix} 1 & 4 & -2 \\ 3 & 2 & -1 \\ 1 & 2 & 5 \end{pmatrix}, \quad A^T \cdot B^T = \begin{pmatrix} 0 & 0 & -6 \\ 3 & 1 & -5 \\ 6 & -4 & 5 \end{pmatrix},$$

$$(A^T)^2 = \begin{pmatrix} 6 & 6 & -3 \\ 8 & 13 & -2 \\ 1 & 8 & 5 \end{pmatrix}.$$

习题 8-5

1. (1) $R(A)=2$; (2) $R(A)=3$; (3) $R(A)=3$.

2.
$$\begin{cases} x_1 = 1 \\ x_2 = 0 \\ x_3 = -1 \\ x_4 = -2 \end{cases}.$$

习题 8-6

1. $A_{11}=2$, $A_{12}=-3$, $A_{13}=2$, $A_{21}=6$, $A_{22}=-6$, $A_{23}=2$, $A_{31}=-4$,
 $A_{32}=5$, $A_{33}=-2$.

$$A^{-1} = \begin{pmatrix} 1 & 3 & -2 \\ -\frac{3}{2} & -3 & \frac{5}{2} \\ 1 & 1 & -1 \end{pmatrix}.$$

2. (1) $A^{-1} = \begin{pmatrix} 1 & -4 & -3 \\ 1 & -5 & 3 \\ -1 & 6 & 4 \end{pmatrix};$

(2) $A^{-1} = \begin{pmatrix} -2 & \frac{2}{3} & -1 \\ -1 & \frac{1}{3} & 0 \\ 2 & -\frac{1}{3} & 1 \end{pmatrix}.$

3.
$$\begin{cases} x = 1 \\ y = 2 \\ z = -2 \end{cases}.$$

测试题八

一、判断题

1. $\times$ 2. $\times$ 3. $\checkmark$ 4. $\times$ 5. $\times$

二、填空题

1.
$$\begin{cases} x = \cos(\beta - \alpha) \\ y = \sin(\beta - \alpha) \end{cases}$$

2. $\begin{vmatrix} 2 & 3 \\ -6 & 5 \end{vmatrix}, (-1)^{2+1} \begin{vmatrix} 2 & 3 \\ -6 & 5 \end{vmatrix}, \begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix}, (-1)^{3+1} \begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix},$

3. $\begin{pmatrix} 7 & 1 \\ -1 & 3 \end{pmatrix}$ 4. $\begin{pmatrix} 1 & 0 \\ -1 & 2 \end{pmatrix}$

5. 行数、列数, 行数、列数.

6. 列数, 行数, A 的行数, B 的列数.

7. $\begin{pmatrix} 6 & 3 \\ 12 & 9 \end{pmatrix}; \begin{pmatrix} 1 & 4 \\ 0 & -1 \end{pmatrix}$

8. $\begin{pmatrix} 0 & -2 & 1 \\ 1 & 0 & 2 \end{pmatrix}$

三、选择题

1. C 2. C 3. A 4. B 5. B
6. C 7. D 8. C 9. B 10. A

四、计算题

1. (1) $-2(x^3 + y^3)$; (2) a^2b^2 ; (3) 1.

2. (1) $\begin{pmatrix} -1 & \frac{1}{4} & \frac{1}{2} \\ \frac{7}{4} & -\frac{1}{8} & -1 \\ -\frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix};$ (2) $\frac{1}{16} \begin{pmatrix} 8 & -4 & 2 & -1 \\ 0 & 8 & -4 & 2 \\ 0 & 0 & 8 & -4 \\ 0 & 0 & 0 & 8 \end{pmatrix}.$

3.
$$\begin{cases} x = -35 \\ y = 30 \\ z = 15 \end{cases}.$$